**Excitations**

So I want to explore the consequences of symmetry apropos the GF’s….

**BCS Model**

We’ll start with the BCS model:



And we considered the following GF’s.



I’d like to verify that these are basically the only independent d.o.f. as far as GF’s go. So I’m going to look at the consequences of various symmetries, and see what they say. To that end, let’s consider more generally,



**Consequence of Translational Symmetry**

From the QM/Identical Partiles/2nd quantization in position space file, we found these properties of momentum creation/annihilation operators.



Let’s verify our H has translational symmetry,



So it does. What consequence does this have for our GF? I guess it means that they must ‘conserve’ momentum. So in other words, consider we had two different momenta (and spins):



And the only way this equation could be true is p = q. Can see that the same considerations would require that F and F† must be defined with q = -p. And so they are.

**Consequence of Spin Rotation Symmetry**

Now let’s consider a 180o rotation of spins,



Checking if H is invariant w/r to this operation:



So we’re good here too. And as for the GF, this implies,



So we have: G↑↑ = G↓↓, and G↑↓ = -G↓↑. And for F we also find: F↑↑ = F↓↓, and F↑↓ = -F↓↑. Same for F†. Now consider π rotations about the x-axis. We found,



and so H would go to:



So it checks out too. And consequence would be:



So we have: G↑↑ = G↓↓, and G↑↓ = G↓↑. Comparing to the consequences of the last rotation, this means we must have G↑↓ = G↓↑ = 0. Doing this with F, we find: F↑↑ = -F↓↓, and F↑↓ = -F↓↑. Comparing to the last rotation, this means we must have: F↑↑ = F↓↓ = 0. Note these restrictions basically mean that spin is conserved. Our GF must create and destroy the same spin, or destroy and destroy opposite spins, or create and create opposite spins.

**Consequence of Parity Symmetry**

What about Parity?



Let’s see if H commutes with the Parity operator.



So as we expect, Parity is a symmetry of our Hamiltonian. And consequence of this on GF? We’ll still allow two different spins…



So that tells us the GF’s are even in their p argument. The same considerations will lead us to conclude that F and F† are also even in their p arguments.

**Consequence of Time Reversal Symmetry**

Finally we’ll consider time-reversal symmetry. Again, from that QM Identical Particles/2nd quantization in momentum space file, we had:



And let’s verify that H has time-reversal symmetry



So it does. And now what are the consequences of this symmetry on the GF’s? Cutting to the chase, we’ll have:



And same for F. This just reproduces earlier results.

**Particle Conservation**

Now of course our Hamiltonian also conserves particle number, since it commutes with the Number operator. But the F GF’s do not conserve particle number, and so they would be zero. So by saying it isn’t zero, it seems we’re actually using, at least implicitly, the Mean Field Hamiltonian. Let’s look at that one.

**Mean Field BCS Model**

So let’s check out the mean field model:



where,



(which I’m going to take to be real, as it seems to turn out that way). And we’ll revisit our GF’s,



And let’s what symmetries H has, and what their implications are:



**Consequence of Translational Symmetry**

I’d like to consider somewhat generally the properties of our GF’s, and specifically, see how many independent GF’s we have. From the 2nd quantization file, we found these properties of momentum creation/annihilation operators.



Let’s verify our H has translational symmetry, going a little faster this time:



So it does, and as before, this mens that momentum must be conserved so that G’s momentum operators must be p and p, while F’s must be p and -p.

**Consequence of Spin Rotation Symmetry**

Now let’s consider a 180o rotation of spins,



Checking if H is invariant w/r to this operation:



So we do have spin rotation symmetry about the y-axis. What about the x-axis?



We have:



As we saw above, the consequences of these two rotations is that: G↑↑ = G↓↓, and G↑↓ = G↓↑ = 0. Whereas, F↑↓ = -F↓↑, and F↑↑ = F↓↓ = 0. Same for F†.

**Consequence of Parity Symmetry**

What about Parity?



Let’s see if H commutes with the Parity operator.



So we do have parity conservation. I had to assume that Δ-k = Δk, but this does actually follow from its form. And so this means the GF’s are symmetric in their p arguments.

So that tells us the GF’s are even in their p argument. The same considerations will lead us to conclude that F and F† are also even in their p arguments.

**Consequence of Time Reversal Symmetry**

Finally we’ll consider time-reversal symmetry.



And let’s verify that H has time-reversal symmetry



So we have TRS. No new consequences…

**General Relationship between F and F†**

Also, from the Statistical Mechanics folder/GF Formal Properties file, we know that:



and also,



So there is only one actually-independent F.